09/19 Lecture 1

Monday, September 19, 2022 4:37 PM

What do you want to get out of/see in math club?

- Operates as a structured "course"
- Higher level STEM academia lectures
- Review lectures (from previous math courses)
- Problem solving within groups
- Applied math- relevant to STEM majors
- Competitions
  - AMATYC https://amatyc.org/page/StudentMathLeague

Lecture - Introduction to Real Analysis

- Proof Definition
- Proofs by Contradiction
- Even-Odd PROOF
- Sets Definition and Properties
- Set Symbols and Examples

What is a Proof?

"If P, then Q" condition

↳ "platonic" role of mathematics

Proofs by Contradiction –

"Suppose *it's* true"

doing some math

it statement → ← *negating it* statement

example: $x = 3 = 4$ (x can NOT equal 3 AND 4 @ the SAME time)

∴ *this* is a contradiction

Even-Odd PROOF

$x, y \in \mathbb{Z}^+ = \{0, 1, 2, \dots, n\}$;

x is even, $x = 2n$; y is odd, $y = 2n+1$

∵ Aside:

$\mathbb{Z}$ symbol for integers

if $x, y \in \mathbb{Z}^-$

⇒ y is still odd BUT $y = 2n - 1$

Prove: $x \cdot y =$ even

PROOF: if x is even, $x = 2n$, and y is odd, $y = 2m+1$;

$\forall n, m \in \mathbb{Z}^+ \cup 0$

$$x \cdot y = 2n(2m+1)$$

$$= \underbrace{2n(2m)}_{\text{even}\cdot\text{even}} + 2n$$

$$= \underbrace{2n2m} + 2n$$

$$= \underset{\text{even}}{2k} + \underset{\text{even}}{2n} = \text{Even} \ \cancel{\emptyset}$$

denotes end of proof

∴ x, y will always be even $\forall n, m$

What are sets?

a set, A, is a collection of objects

denoted by $\{\_\}$ → inside curly braces are your elements

∵ Aside

objects can be any "thing"

↳ in math, objects are #s, points, functions, etc.

properties

– let $A = B$ so, $A = \{a_1, a_2, \dots, a_n\}$

$B = \{b_1, b_2, \dots, b_n\}$

$\Rightarrow A = B$ iff $a_i = b_i$ $\forall$ elements in A, B

"if and only iff"

– let $A = \{a_1, a_2, \dots, a_j, \dots, a_{n-1}, a_n\}$

$:= \{a_1, a_2, \dots, a_j, \dots, a_{n-1}, a_n\}$

∵ Aside

:= treated as an assignment (similar to using one = symbol rather than two == symbols in some CP langauges)/ an asymmetrical relationship

Set Symbols/Notation

– Empty Set: $\emptyset = \{\ \}$

$\{\emptyset\} = \{\{\ \}\}$

– Singleton Set: a set containing ∠ object

ex: $\{1\}$

– Subset: let A, B be sets

$A \subset B$

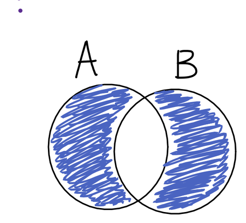
inclusion sign OR ⇒ A is a subset of B

$A \subseteq B$

– Union of two sets:

$A \cup B$

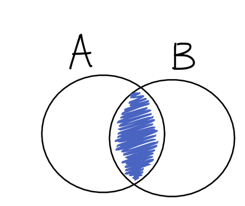
"OR"



– Intersection:

$A \cap B$

"AND"



– Cardinality:

$|A|$ = "magnitude/size of" A ⟷ # of elements in A

examples:

– $|\{1, 3\}| = 2$ – $|\emptyset| = 0$

– $|\mathbb{N}| = \{1, 2, 3, \dots\} = \aleph_0$

– $|\mathbb{R}|$ = uncountable/ nondenumerable (infinite set)

∵ Aside

– $\mathbb{N}$ symbol for natural #s

– $\aleph_0$ "Aleph Null" symbolizes the size of an infinite collection of objects w/ an infinite amount of elements ⇒ a continuum

– $\mathbb{R}$ symbol for Real #s